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LAMINAR FILM CONDENSATION OF A CHEMICALLY REACTING GAS

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Abstract—A theoretical study is made of laminar film condensation of a chemically reacting gas with free convection heat and mass transfer. Numerical results are obtained for the particular system $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO+O_2$. A chemical reaction between noncondensable components, $2NO+O_2 \rightleftarrows 2NO_2$, is shown to increase the rate of heat and mass transfer in comparison with the "frozen" case.

Based on the theoretical analysis, the criterial equation is derived, the use of which has allowed treatment and correlation of the numerical and experimental data on condensation of N_2O_4 of nonequilibrium composition.

NOMENCLATURE

c_p,	specific heat at constant pressure;
D_k,	diffusion coefficient of component k;
h_{fg},	latent heat of condensation;
I_k,	mass source (sink) of component k due to chemical reactions;
i,	specific enthalpy of a gas system, $= \sum_{k=1}^{k=n} m_k i_k$;
i_k,	specific enthalpy of component k;
j_k,	diffusive mass flux of component k;
k,	thermal conductivity;
K_c,	equilibrium constant;
K_{∂},	rate constant for the reaction of dissociation;
K_r,	rate constant for the reaction of recombination;
m_k,	mass fraction of component k;
M,	molecular weight;
Pr,	Prandtl number, $= \nu c_p \rho/k$;
R,	gas constant;
Sc_k,	Schmidt number of component k, $= \nu/D_k$;
T,	absolute temperature;
u, v,	velocity components;
x, y,	boundary-layer coordinates.

Greek symbols

δ,	condensate film thickness;
μ,	absolute viscosity;
ν,	kinematic viscosity;
ρ,	density.

Subscripts

f,	"frozen";
k,	component k;
l,	values in the liquid phase;
max,	maximum value;
s,	values at the saturation line;
w,	values at the wall;
δ,	values at the interface;
∞,	values in the bulk region;
I,	first reaction $N_2O_4 \rightleftarrows 2NO_2$;
II,	second reaction $2NO_2 \rightleftarrows 2NO+O_2$;
1,	component N_2O_4;
2,	component NO_2;
3,	component NO;
4,	component O_2.

INTRODUCTION

THE PRESENT paper is concerned with condensation of a gas containing both condensable and noncondensable components that enter into chemical homogeneous reactions. An example of such a gas is provided by the chemically reacting system $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO+O_2$, which appears to offer considerable promise as a coolant of the single-loop nuclear power stations [1–3]. When being cooled, this system undergoes two recombination reactions in succession. The first reaction, $2NO+O_2 \rightleftarrows 2NO_2$, proceeds rather slowly and the time required for the establishment of equilibrium, depending on thermodynamic parameters, is 10^{-4}–0.1 s, while the relaxation time of the chemical reaction $2NO_2 \rightleftarrows N_2O_4$ is 10^{-6}–10^{-8} s and this reaction is, in fact, in equilibrium over the whole range of parameters.

This general problem involves two limiting cases: when composition of the mixture is "frozen", i.e. the rates of chemical reactions are zero, there is condensation of an ordinary vapour–gas mixture, and, on the other hand, when the reaction $2NO+O_2 \rightleftarrows 2NO_2$ is completed, an "equilibrium" case, there is condensation of a pure vapour consisting of NO_2 and N_2O_4 only. Both cases are analyzed theoretically in detail in [4–10]. Some of their results have been used in the present paper.

ANALYSIS

Governing equations and boundary conditions

The problem at hand is the classical problem of condensation on a vertical plate exposed to a large body of quiescent vapour with a condensate film flowing downwards under gravity. According to the common approach, the system of the initial equations is formulated in the approximations of the boundary-layer theory. For the gas boundary layer adjacent to the condensate film, the system of differential equations for heat and mass transfer in a general case is of the form:

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=(\rho-\rho_\infty)g+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{1}$$

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \tag{2}$$

$$\rho u\frac{\partial i}{\partial x}+\rho v\frac{\partial i}{\partial y}=\frac{\partial}{\partial y}(-q_{\text{eff}}) \tag{3}$$

$$\rho u\frac{\partial m_k}{\partial x}+\rho v\frac{\partial m_k}{\partial y}=\frac{\partial}{\partial y}(-j_k)+I_k. \tag{4}$$

In the energy equation (3) transition from enthalpies to temperatures is performed in two ways. When composition of a reacting gas corresponds to an essentially nonequilibrium state, the use of the equation for total energy flux [11]

$$q_{\text{eff}}=-\lambda_f\frac{\partial T}{\partial y}+\sum_{k=1}^{k=n} i_k j_k$$

and of the diffusion equation (4) reduces equation (3) to:

$$\rho c_{pf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_f\frac{\partial T}{\partial y}\right)+\sum_{k=1}^{k=n}(c_{pk}j_k)\frac{\partial T}{\partial y}+\sum_{k=1}^{k=n} i_k I_k, \tag{3a}$$

where c_{pf} and λ_f are the "frozen" specific heat and thermal conductivity of the mixture, and c_{pk} is the "frozen" specific heat of the component k.

Under equilibrium conditions, the energy equation has a more simple form, since the last two terms on the RHS of equation (3a) may be discarded, and heat of reactions and diffusion thermo may be taken into account by the "effective" specific heat and thermal conductivity.

The present paper deals with the following situation. A chemically reacting gas, $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO + O_2$, is at a temperature and pressure near the saturation line. In the main, the gas is composed of an equilibrium mixture of the N_2O_4 and $2NO_2$ molecules ($N_2O_4 \rightleftarrows 2NO_2$) involving a small amount (up to several percent) of the NO and O_2 molecules which have not managed to recombine. The calculations performed have shown that under these conditions the thermophysical properties of the gas (specific heat, thermal conductivity, viscosity, etc.) differ slightly from the "effective" ones, which correspond to the equilibrium gas state at the given temperatures and pressures. Besides, superheating of the gas phase in the present problem being small (up to ten degrees in the main variants), the temperature dependence of the thermophysical variables may be ignored and they can be taken constant. However, as has been shown in [7], with noncondensable gases, even a small initial temperature difference gives rise to buoyancy forces in the gas phase. Therefore, it is necessary to retain a variable density on the RHS of equation (1), which is the practice in the solution of the boundary-layer problems with natural convection [12]. So the system of equations of momentum, continuity, energy and diffusion may be written as:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\left(1-\frac{\rho_\infty}{\rho}\right) \tag{1a}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2a}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{c_p\rho}\frac{\partial^2 T}{\partial y^2} \tag{3b}$$

$$u\frac{\partial m_k}{\partial x}+v\frac{\partial m_k}{\partial y}=D_k\frac{\partial^2 m_k}{\partial y^2}+\frac{I_k}{\rho}. \tag{4a}$$

The temperature dependence of the gas density was given as $\rho=\rho_\infty[1+\beta(T-T_\infty)]$, where the thermal expansion coefficient is calculated with the aid of the state equation [13]. Equation (4) is written for the noncondensable component O_2. In the "frozen" case, when noncondensable components do not take part in chemical reaction, $I_k=0$.

In a general case, the system of equations for a liquid boundary layer should be similar to (1)–(4). However, for the specific conditions under consideration we can make some simplifications. When the system $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO+O_2$ condenses, the liquid phase, composed of the N_2O_4 and NO_2 molecules only, can be treated as a uniform substance which permits the diffusion equation to be excluded from the analysis. Moreover, following [7, 9, 10], we shall ignore the inertia and convection terms in the equations of momentum and energy. Thus, the differential equations for the liquid phase may be written as:

$$g+\nu_l\frac{\partial^2 u_l}{\partial y^2}=0 \tag{5}$$

$$\frac{\partial u_l}{\partial x}+\frac{\partial v_l}{\partial y}=0 \tag{6}$$

$$\frac{\partial^2 T_l}{\partial y^2}=0. \tag{7}$$

Following the results of [8], thermophysical properties of the condensate were assumed constant and evaluated at the reference temperature $T^*=T_w+0.31(T_s-T_w)$.

Equations (1)–(7) are supplemented by the following boundary conditions:

at $y=0$

$$T_l(x,0)=T_w,\quad u_l=v_l=0, \tag{8}$$

at $y=\delta(x)$

$$T=T_l=T_s;\quad u=u_l;\quad \mu\frac{\partial u}{\partial y}=\mu_l\frac{\partial u_l}{\partial y} \tag{9}$$

$$\rho\left(u\frac{\mathrm{d}\delta}{\mathrm{d}x}-v\right)=\rho_l\left(u_l\frac{\mathrm{d}\delta}{\mathrm{d}x}-v_l\right) \tag{10}$$

$$k_l\frac{\partial T_l}{\partial y}=k\frac{\partial T}{\partial y}+\rho h_{fg}\left(u\frac{\mathrm{d}\delta}{\mathrm{d}x}-v\right) \tag{11}$$

$$m_k\left(u\frac{\mathrm{d}\delta}{\mathrm{d}x}-v\right)-D_k\frac{\partial m_k}{\partial y}=0, \tag{12}$$

at $y \to \infty$

$$u(\infty)=0,\quad T(\infty)=T_\infty,\quad m_k(\infty)=m_{k\infty}. \tag{13}$$

The system of equations (1)–(13) for a chemically reacting gas is closed both by the kinetic equations for the relationship between concentrations of the components and by the equation for the intensity of an inner mass source. For the $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO + O_2$ system

$$m_1=\frac{M_1}{M_2^2}\frac{\rho}{K_{cI}}m_2^2,\quad m_3=2\frac{M_3}{M_4}m_4,\quad \sum_{k=1}^{k=4}m_k=1 \tag{14}$$

$$I_4=\frac{M_4}{M_2^2}K_{\partial II}\rho m_2^2-\frac{K_{rII}}{M_3^2}\rho m_3^2 m_4. \tag{15}$$

In equations (4) and (12) oxygen is chosen as a noncondensable component, since it takes part only in one reaction, and by it the concentrations of other components can be most simply expressed.

METHOD OF SOLUTION

Numerical solution of the problem is much eased if, following [7], the initial system of differential equations is subjected to the following similarity transformations.

Introduce a dimensionless coordinate

$$\eta=y\sqrt[4]{\left(\frac{g}{4v^2x}\right)} \tag{16}$$

which in the liquid phase changes from 0 to η_δ and in the gas phase, from 0 to ∞, with the coordinate η_δ in the liquid corresponding to $\eta=0$ in the gas.

If, as usual, the stream function is defined by

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x},$$

then dimensionless stream functions for the gas, $F(\eta)$, and for the liquid, $f(\eta)$, phases may be expressed as:

$$F(\eta)=\frac{\psi}{4cvx^{0.75}},\quad f(\eta)=\frac{\psi_l}{4c_l v_l x^{0.75}}, \tag{17}$$

where

$$c=\left(\frac{g}{4v^2}\right)^{0.25},\quad c_l=\left(\frac{g}{4v_l^2}\right)^{0.25}.$$

Upon introducing dimensionless temperatures

$$\theta=\frac{T}{T_\infty},\quad \vartheta=\frac{T_l}{T_\infty} \tag{18}$$

and using (16)–(18), the initial partial differential equations (1a)–(7) may be reduced to ordinary differential equations:

for the gas phase

$$F'''+3FF'-2(F')^2+\beta T_\infty(\theta-1)=0 \tag{19}$$

$$m_k''+3m_k'F'Sc_k+2I_kSc_k\left(\frac{x}{g}\right)^{0.5}\frac{1}{\rho}=0 \tag{20}$$

$$\theta''+3PrF\theta'=0, \tag{21}$$

for the liquid phase

$$f'''+1=0 \tag{22}$$

$$\vartheta''=0. \tag{23}$$

In terms of the transformed variables the boundary conditions become:

on the condensation surface ($\eta=0$)

$$f(0)=0,\quad f'(0)=0,\quad \vartheta(0)=\vartheta_w, \tag{24}$$

at the interface

$$\theta(0)=\vartheta(\eta_\delta),\quad F'(0)=f'(\eta_\delta),\quad F''(0)=Rf''(\eta_\delta) \tag{25}$$

$$F(0)=Rf(\eta_\delta)+\tfrac{4}{3}x\frac{\mathrm{d}\eta_\delta}{\mathrm{d}x}f'(\eta_\delta)(R-1) \tag{26}$$

$$\theta'(0)=\left\{\frac{k_lc_l}{\eta_\delta}[\vartheta(\eta_\delta)-\vartheta_w]-\frac{3c\mu h_{fg}}{T_\infty}\left[F(0)+\tfrac{4}{3}xF'(0)\frac{\mathrm{d}\eta_\delta}{\mathrm{d}x}\right]\right\}\frac{1}{kc} \tag{27}$$

$$m_k'(0)+3m_k(0)Sc_k\left[F(0)+\tfrac{4}{3}x\frac{\mathrm{d}\eta_\delta}{\mathrm{d}x}F'(0)\right]=0, \tag{28}$$

in the gas bulk region ($\eta \to \infty$)

$$F'(\infty)=0,\quad \theta(\infty)=1,\quad m_k(\infty)=m_{k\infty}. \tag{29}$$

In formulae (25)–(26) $R=(\rho_l\mu_l/\rho\mu)^{0.5}$.

The analysis of equations (19)–(29) shows that the solution cannot be similar, since the coordinate x enters in equation (20) and in boundary conditions (26)–(28) as a parameter. For this very reason, the dimensionless velocity, temperature and concentration will have different profiles at each of the locations along the x axis. However, solution of the problem is much eased if the derivative $\mathrm{d}\eta_\delta/\mathrm{d}x$ is set equal to zero. Then

$$\eta_\delta=\delta\left(\frac{g}{4v_l^2x}\right)^{0.25}, \tag{30}$$

which corresponds to the assumption that the condensate film has a Nusselt profile, i.e. $\delta=\text{const.}\,x^{0.25}$.

It has been shown in [14] that in the case of condensation of quiescent vapour on a vertical plate surface, the profiles of the condensate film differ but slightly for different boundary conditions on the wall of the first, second or the third kind, with the film profiles at the boundary conditions of the first kind (i.e. $T_w=\text{const.}$) and of the third kind (i.e. with specified heat-transfer coefficient and temperature of the cooling medium) practically coinciding. Thus, in real conditions, the thickness of the condensate film obeys the law close to $\delta=\text{const.}\,x^{0.25}$. Incidentally, this approach of specifying the film profile was used by F. S. Felicione and R. A. Seban who investigated vapour condensation on a vertical plate in the presence of noncondensable but soluble gases [15].

In this case the temperature of the surface, where condensation takes place, will not be isothermal and it should be found from the solution of the problem, but then the boundary conditions (26)–(28) assume the following form:

$$F(0) = Rf(\eta_\delta) \tag{26a}$$

$$\theta'(0) = \frac{k_l c_l}{kc}(\vartheta(\eta_\delta)-\vartheta_w) - \frac{3c\mu h_{fg}}{kcT_\infty}F(0) \tag{27a}$$

$$m_k'(0)+3m_k(0)Sc_k F(0) = 0. \tag{28a}$$

The system of equations (14), (15), (19)–(25), (26a)–(28a), (29) has been solved on the computer "Minsk-22" by the numerical method described in [14].

In the "frozen" case (when $I_4 = 0$), the solutions to the differential equations are similar, i.e. the dimensionless profiles of temperatures, velocities and concentrations are independent of x; the parameters $T_w(x)$, $T_s = T_\delta(x)$ and $m_{k\delta}(x)$ remain constant.

The choice of the computational technique with the use of condition (30) has appeared to be successful, because prediction of all the unknown functions at any x does not depend on the results of the previous steps. This permits the calculations to be made with any step and offers means of reducing computer time considerably.

ANALYSIS OF NUMERICAL RESULTS

The initial quantities are taken to be pressure, bulk temperature of the gas, concentration of the noncondensable components, dimensionless film thickness η_δ and the maximum height X of the vertical plate. The last parameter was defined on the basis of the condition of preserving a laminar flow regime of the film condensate. The critical Reynolds number $Re = 4\delta u_l/\nu_l$, which corresponds to the onset of a wavy flow, was taken to be equal to 40 according to [17].

The main calculations were performed for the pressures of 1 and 1.6 bar. Thermophysical and transport properties were taken from [13], the rate constants for chemical reactions of dissociation and recombination were determined as:

$$K_{\partial II} = 10^{8.97}\exp\left(-\frac{25\,800}{RT}\right),$$

$$K_{rII} = \frac{10^{1.32}\exp(960/RT)}{2\dfrac{M_2 m_4}{M_4 m_2} + 10^{-1.55}\exp(-1500/RT)}. \tag{31}$$

Integration of equations (22) and (23) for a liquid phase gives a linear temperature profile and the profiles of velocity which differ but slightly from the Nusselt profile [18]. For revealing the mechanism of heat and mass transfer at condensation of a chemically reacting gas, the phenomena in a gas boundary layer adjacent to the condensate film are of great interest.

Figure 1 presents distributions of the velocity in a longitudinal direction and in one of the locations across the gas boundary layer. The dashed lines correspond to the "frozen" case, when there are noncondensable gases in the mixture not participating in chemical reaction, while the solid lines refer to the case when kinetics of the chemical reaction $2NO + O_2 \rightleftarrows 2NO_2$ is taken into account. It is the character of the velocity distribution which attests to the fact that free convection predominates in the convective transfer processes occurring in the gas boundary layer. For comparison, curve 4 in Fig. 1 gives the velocity profile of a pure saturated condensing vapour without allowance for free convection, with remainder parameters being equal. In this case the derivative $\partial u/\partial y$ at the interface changes its sign to the opposite.

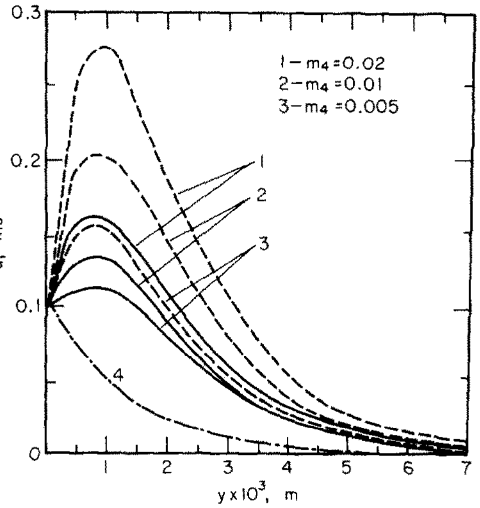

FIG. 1. Streamwise velocity component distribution in the gas boundary layer at $x = 0.37$ m, $\eta_\delta = 0.23$.

The profiles of temperatures and concentrations of the noncondensable component NO_2 in a gas phase are displayed in Fig. 2, which shows clearly that the chemical reaction between noncondensing components levels off their temperature and concentration profiles. An essential point is that in the "kinetic" case the concentration of a noncondensable gas at the

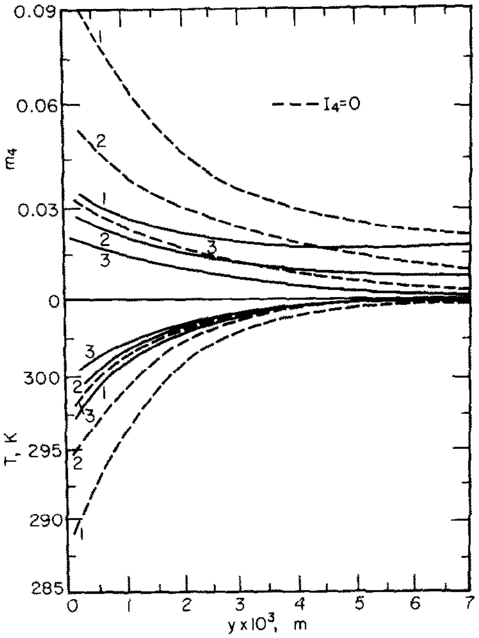

FIG. 2. Temperature and concentration profiles of the noncondensable component O_2 in the gas boundary layer (notations of Fig. 1 are employed here).

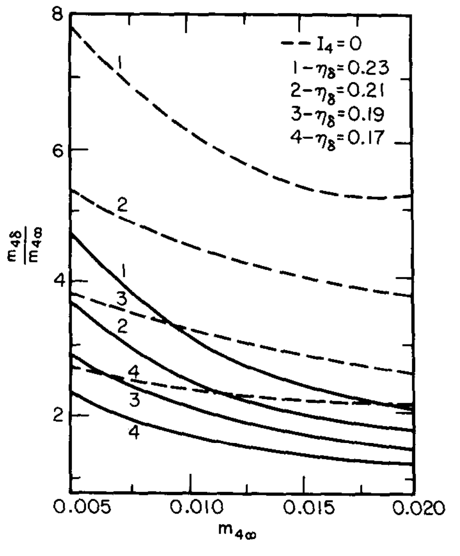

FIG. 3. Ratio of noncondensable gas concentration at the interface to that in the bulk region at different condensation intensities.

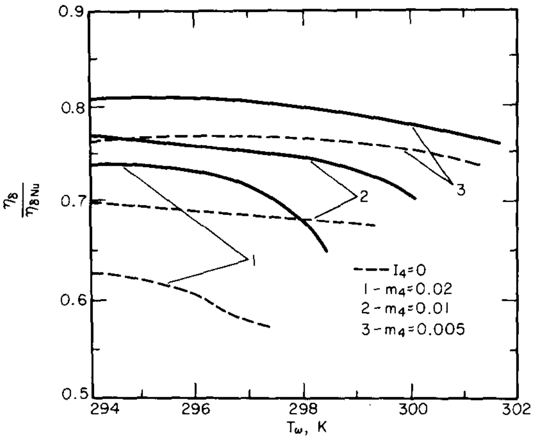

FIG. 4. Relative condensation intensity vs mean wall temperature.

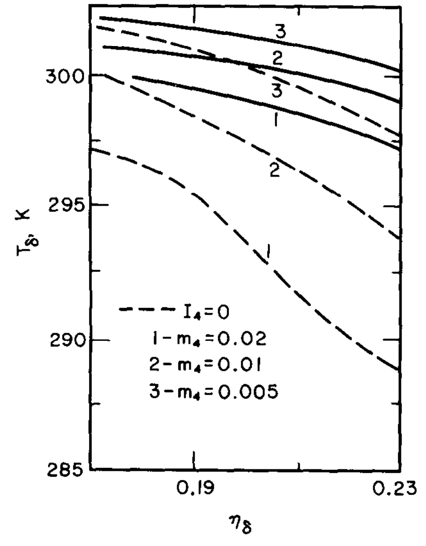

FIG. 5. Condensation intensity effect on interfacial temperature.

interface is lower than in the "frozen" case, which improves heat transfer in the process of condensation.

A favourable effect of the chemical reaction is more evident from Fig. 3, where $m_{4\delta}/m_{4\infty}$ is given vs bulk concentration of a noncondensable gas.

The character of condensation heat transfer of a chemically reacting gas depends on the relation between the three "competing" but interconnected processes: mass exchange (diffusive and convective) between the interface and the bulk of the gas; chemical reaction, by which noncondensable components are transformed into condensable ones; and condensation itself. From the phenomenological viewpoint, the intensity of condensation is more clearly characterized by the dimensionless parameter η_δ defined by relation (30). Figure 4 gives the numerical results for the ratio $\eta_\delta/\eta_{\delta Nu}$, where $\eta_{\delta Nu}$ is the condensation intensity parameter evaluated as a function of the mean wall temperature by using the Nusselt model. These data show that a reaction between noncondensable components promotes an essential increase in the condensation intensity. Figure 5 illustrates variation of the interfacial temperature with the condensation intensity parameter. The analysis of this situation is of great practical importance. Just suppose, that it is necessary to increase the condensation intensity by outer effect (for example, by increasing the heat-transfer coefficients of the surrounding medium or by decreasing its temperature). As follows from Fig. 5, the increase in condensation intensity (i.e. dimensionless film thickness) leads to an opposite phenomenon, i.e. to an increase in concentration of condensable components at the interface and a decrease in a condensation temperature. However, the kinetics of a chemical reaction noticeably reduces this unfavourable effect.

CORRELATION OF NUMERICAL AND EXPERIMETAL DATA

Of paramount importance for the practical purposes is correlation of the predicted and measured results in a usable criterial form. As far as the authors know, to date two contributions have been published on condensation of the chemically reacting gas $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO+O_2$ [19, 20], but they differ in the methods of treatment of experimental data. Thus in [19] the interpretation of the data has been based on the heat-transfer coefficient. However, at present in the works on condensation in the presence of noncondensable gases, use is more frequently made of the mass-transfer coefficient which characterizes the rate of transport of condensable components (vapour) to the interface. For a chemically reacting gas, calculation of the total mass flux (convective and diffusive) of all the condensable components is very complicated, while the condition of impermeability of noncondensable components (12) provides a good means for determining the desired quantity. If the concept of the mass-transfer coefficient is introduced and defined as:

$$\beta_{Dk} = -\frac{\rho D_k \operatorname{grad} m_{k/\delta}}{m_{k\delta} - m_{k\infty}}, \qquad (32)$$

then condition (12) somewhat differs from the traditional form of the equation for mass flux as

$$j = \frac{\beta_{Dk}(m_{k\delta} - m_{k\infty})}{m_{k\delta}}. \tag{33}$$

Though by definition the quantity β_{Dk} denotes only the diffusional transfer of the noncondensable component, the quantity j in equation (33) is the total mass flux of the condensable substance (diffusional and convective). Thus the problem is reduced to the search for an equation for calculation of the mass transfer coefficient of one of the noncondensable components. For this purpose we shall use the method of "float-and-sink" analysis of the governing differential equations and boundary conditions [21]. In more detail the procedure for the condensation problems is described in [22].

Transform the system of equations (1)–(13) to a dimensionless form using new variables which change from 0–1:

$$\tilde{u} = \frac{u}{u_{\max}}, \quad \tilde{v} = \frac{v}{v_{\max}}, \quad \tilde{x} = \frac{x}{X}, \quad \tilde{\delta} = \frac{\delta}{\delta_{\max}}, \tag{34}$$
$$\tilde{m}_k = \frac{m_k - m_{k\infty}}{m_{k\delta} - m_{k\infty}}, \quad \tilde{T}_l = \frac{T_l - T_w}{T_s - T_w}, \quad \tilde{T} = \frac{T - T_s}{T_\infty - T_s}$$

To determine the dimensionless coordinate $\tilde{y}$, different scale parameters are chosen depending on the type of the equation, which involves this quantity. In this diffusion equation $\tilde{y} = y/\delta_D$ and in the momentum equation $\tilde{y} = y/\delta_m$, where δ_D and δ_m are the thicknesses of the diffusional and dynamic boundary layer, respectively.

The values of maximum velocity components, $u_{\max}$ and $v_{\max}$, are not known. However, the orders of these quantities can be easily determined on reducing the governing equations to a dimensionless form. Thus, for example, continuity equation (2a) may be transformed to:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{v_{\max} X}{u_{\max}\delta_m}\frac{\partial \tilde{v}}{\partial \tilde{y}} = 0. \tag{2b}$$

Since by definition, all new dimensionless variables are of the same order (vary from 0 to 1), equation (2b) is valid if

$$\frac{v_{\max} X}{u_{\max}\delta_m} \sim 1, \quad \text{or} \quad v_{\max} \sim u_{\max}\frac{\delta_m}{X}. \tag{35}$$

Having normalized boundary condition (11) in a similar fashion and taken into account (35), we have:

$$u_{\max} \sim \frac{k_l(T_s - T_w)X}{\rho_l h_{fge}\delta_{\max}\delta_m} \tag{36}$$

where h_{fge} is the "effective" latent heat of condensation with regard for superheat in the gas phase.

This procedure of estimation with reduction of the system of equations (1a)–(13) to a dimensionless form gives the following complexes that characterize condensation mass transfer of a chemically reacting gas:

$$Ar = \frac{g(\rho - \rho_{\infty})X^3}{\rho v^2}, \quad Re_T = \frac{k_l(T_s - T_w)X}{\delta_{\max}\mu_l h_{fge}}, \quad Sc_k = \frac{v}{D_k}$$

$$Da_k = \frac{I_k X^2}{\rho D_k}, \quad \Delta m_k = m_{k\delta} - m_{k\infty}, \quad \left(\frac{\delta_m}{\delta_D}\right),$$
$$\left(\frac{\mu_l}{\mu}\right), \quad Nu_{Dk} = \frac{\beta_{Dk}X}{\rho D_k}, \quad \beta_{Dk} = \frac{\rho D_k}{\delta_D}.$$

But not all of these parameters are independent of each other. Thus it is easily seen that $(\delta_m/\delta_D) \sim Sc_k$ while the quantity $m_{k\delta}$ is related to the temperature T_s by the thermodynamic condition on the saturation line $T_s = f(m_{k\delta})$. The resulting criterial equation for calculation of the mass-transfer coefficient at condensation of a chemically reacting gas is as follows:

$$Nu_{Dk} = \text{const.}\, Ar^{n_1} Re_T^{n_2} Sc_k^{n_3} m_{k\infty}^{n_4} Da_k^{n_5}\left(\frac{\mu_l}{\mu}\right)^{n_6}. \tag{37}$$

Equation (37) has a clear physical meaning. The terms on the RHS show the effect of different factors on mass transfer in a gas phase, namely, free convection, Ar number; outer heat flux (surrounding cooling intensity), Re_T; transport properties of a gas, Sc_k; concentrations of a noncondensable gas, $m_{k\infty}$; kinetics of a chemical reaction, Da_k; interfacial shear, simplex (μ_l/μ). It is very important that the mass-transfer intensity depends directly on the thermal number, Re_T, which is indicative of interrelation between the heat- and mass-transfer processes.

Equation (37) has been used to interpret the numerical data obtained by the above theory. Over the range of parameters $P = 1$–1.6 bar; $Ar = (5–53)\cdot 10^8$; $Re_T = 4$–18, $m_{4\infty} = 0.005$–0.02; $Da_4 = 0.5$–69, accurate to 6%, the numerical data are correlated by

$$Nu_{D4} = 3.11\cdot 10^{-3} Ar^{0.25} Re_T^{0.4} Sc_4^{0.75} \times m_{4\infty}^{-0.37} Da_4^{0.2}\left(\frac{\mu_l}{\mu}\right). \tag{37a}$$

Here T_∞ has been taken as a reference temperature and the height of the vertical wall, as a characteristic dimension.

This very equation has been employed to treat the experimental data of [19] on condensation of the chemically reacting gas $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO + O_2$ on a horizontal tube (Fig. 6). For pressures of 1.5–8 bar, heat fluxes $q = (1–18)\cdot 10^4\ W/m^2$, degrees of dissociation of the NO_2 molecules in the condenser volume from 1 to 35%, these data are correlated,

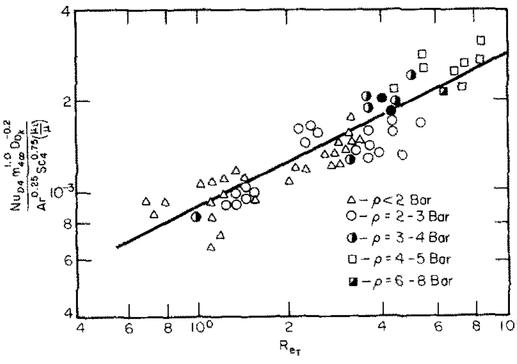

FIG. 6. Correlation of experimental data on condensation mass transfer of a chemically reacting gas.

accurate to 8% (mean square deviation with reliability coefficient of 0.95), by the equation

$$Nu_{D4} = 0.88 \cdot 10^{-3} Ar^{0.25} Re_T^{0.5} Sc_4^{0.75} \times m_{4\infty}^{-1.0} Da_4^{0.2} \left(\frac{\mu_l}{\mu}\right) \quad (37b)$$

where the tube diameter is taken as a characteristic dimension.

An increase in the exponent of $m_{4\infty}$ in equation (37b), as compared to equation (37a), is probably due to transition from the plane problem to the cylindrical one and may be interpreted as an increase in the diffusional resistance caused by contraction of the front with radial supply to the condensation surface on a horizontal tube.

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CONDENSATION EN FILM LAMINAIRE D'UN GAZ EN REACTION CHIMIQUE

Résumé—On étudie théoriquement la condensation en film laminaire d'un gaz en réaction chimique avec convection naturelle de chaleur et de masse. On obtient des résultats numériques pour le système particulier $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO+O_2$. Une réaction chimique entre les composants incondensables, $2\,NO+\frac{1}{2}O_2 \rightleftarrows 2\,NO_2$, accroît le transfert de chaleur et de masse, par rapport au cas gelé.

A partir de l'analyse théorique, l'equation critique est obtenue pour obtenir une corrélation entre les valeurs numériques et expérimentales sur la condensation de $N_2\ O_4$ dans une composition non équilibrée.

LAMINARE FILMKONDENSATION EINES CHEMISCH REAGIERENDEN GASES

Zusammenfassung—Die laminare Filmkondensation eines chemisch reagierenden Gases mit Wärmeübertragung durch freie Konvektion und Stoffübertragung wurde theoretisch untersucht. Numerische Ergebnisse wurden erzielt für das System

$$N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO+O_2.$$

Es wird gezeigt, daß eine chemische Reaktion zwischen nichtkondensierbaren Komponenten, 2NO+

$O_2 \rightleftarrows 2NO_2$, den Wärme und Stoffübergang im Vergleich zu dem "eingefrorenen" Fall erhöht. Aufgrund der theoretischen Analyse wird eine Kriteriumsgleichung abgeleitet. Sie erlaubt die Behandlung und Korrelation der numerischen und experimentellen Ergebnisse von kondensierendem N_2O_4 bei Nichtgleichgewichtszusammensetzung.

ЛАМИНАРНАЯ ПЛЕНОЧНАЯ КОНДЕНСАЦИЯ ХИМИЧЕСКИ РЕАГИРУЮЩЕГО ГАЗА

Аннотация — Представлено аналитическое исследование характеристик тепло- и массопереноса при ламинарной пленочной конденсации химически реагирующего газа в условиях свободной конвекции. Численное решение получено для конкретной системы $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO + O_2$. Показано, что химическая реакция между неконденсируемыми компонентами $2NO + O_2 \rightleftarrows 2NO_2$ способствует увеличению интенсивности процессов тепло- и массопереноса по сравнению с «замороженным» случаем.

На основе теоретического анализа получено критериальное уравнение, с помощью которого обработаны и обобщены результаты численных расчетов и экспериментальных данных по конденсации N_2O_4 неравновесного состава.